Egyváltozós analízis differenciálszámítás gyakorló feladatok megoldása

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(1) $(2+x-x^2)' = 1-2x$

(2) $(a^5+5a^3x^2-x^5)' = 10a^3x-5x^4$

(3) $((x-a)(x-b))' = 1\cdot(x-b)+(x-a)\cdot 1 = 2x-a-b$

(4) $\left(\frac{2x}{1-x^2}\right)' = \frac{2(1-x^2)+4x^2}{(1-x^2)^2}$

(5) $\left(\sqrt{x+\sqrt{x}}\right)' = \frac{1}{2}\frac{1}{\sqrt{x+\sqrt{x}}}\cdot\left(1+\frac{1}{2}\frac{1}{\sqrt{x}}\right)$

(6) $(x+\sqrt{x}+\sqrt[3]{x})' = 1+\frac{1}{2}\cdot\frac{1}{\sqrt{x}}+\frac{1}{3}\cdot\frac{1}{\sqrt[3]{x^2}}$

(7) $(\cos 2x - 2\sin x)' = -(\sin 2x)\cdot 2 - 2\cdot\cos x$

(8) $((x\sin\alpha+\cos\alpha)(x\cos\alpha-\sin\alpha))' = \sin\alpha(x\cos\alpha-\sin\alpha)+\cos\alpha(x\sin\alpha+\cos\alpha) = x\sin 2\alpha+\cos 2\alpha$

(9) $\left(\frac{\sin^2 x}{\sin x^2}\right)' = \frac{2\sin x\cos x\sin x^2 - 2x\sin^2 x\cos x^2}{\sin^2 x^2}$

(10) $\left(e^{-x^2}\right)' = \left(e^{-x^2}\right)\cdot(-2x)$

$$(11)\qquad (\ln\ln\ln x)' = \frac{1}{\ln\ln x}\cdot\frac{1}{\ln x}\cdot\frac{1}{x}$$

$$(12)\qquad \left(\ln\operatorname{tg}\tfrac{x}{2}\right)' = \frac{1}{\operatorname{tg}\frac{x}{2}}\cdot\frac{1}{\cos^2\frac{x}{2}}\cdot\frac{1}{2}$$

$$(13)\qquad \left(\frac{\sin(2x)}{\sin(2x)+\cos(2x)}\right)' = \frac{2\cos(2x)(\sin(2x)+\cos(2x)) - \sin(2x)(2\cos(2x)-2\sin(2x))}{(\sin(2x)+\cos(2x))^2}$$

$$(14)\qquad \left(\sqrt{\ln(\cos(x+1))}\right)' = \frac{1}{2}\left(\ln(\cos(x+1))\right)^{-\frac{1}{2}}\frac{1}{\cos(x+1)}(-\sin(x+1)) = \frac{-\operatorname{tg}(x+1)}{2\sqrt{\ln(\cos(x+1))}}$$

$$(15)\qquad \left((1-x)\operatorname{arc\,tg}(x^2)\right)' = -\operatorname{arc\,tg}(x^2) + (1-x)\frac{1}{1+(x^2)^2}2x$$

$$(16)\qquad \left(\operatorname{arc\,tg}\left(\frac{1}{x}\right)\right)' = \frac{1}{1+\left(\frac{1}{x}\right)^2}(-1)x^{-2} = \frac{-1}{x^2+1}$$

$$(17)\qquad \left(\sqrt[3]{\sqrt{x^3}\sqrt{x}}\right)' = \left(\sqrt[3]{x^2}\right)' = \frac{2}{3}x^{-\frac{1}{3}} = \frac{2}{3\sqrt[3]{x}}$$

$$(18)\qquad \left(x^3e^{\sin(3x)}\right)' = 3x^2e^{\sin(3x)} + x^3e^{\sin(3x)}3\cos(3x)$$

$$(19)\qquad \left(\sqrt[4]{\ln(x+2)+4\cos\left(\frac{x}{2}\right)}\right)' = \tfrac{1}{4}\left(\ln(x+2)+4\cos\left(\tfrac{x}{2}\right)\right)^{-\frac{3}{4}}\left(\tfrac{1}{x+2}+4\cdot\tfrac{1}{2}\left(-\sin\left(\tfrac{x}{2}\right)\right)\right)$$

$$(20)\qquad \left(\ln\left(\frac{x}{3}\right)^2\right)' = \left(\ln\left(\left(\frac{x}{3}\right)^2\right)\right)'\left(2\ln\left(\frac{x}{3}\right)\right)' = \frac{2}{x}$$

$$(21)\quad \left(\ln^2\left(\frac{x}{3}\right)\right)' = \left(\left(\ln\left(\frac{x}{3}\right)\right)^2\right)' = 2\ln\left(\frac{x}{3}\right)\frac{1}{x}$$

$$(22)\quad \left(\frac{1}{\sqrt{x}e^{-x^2}}\right)' = \left(x^{-\frac{1}{2}}e^{x^2}\right)' = -\frac{1}{2}x^{-\frac{3}{2}}e^{x^2} + x^{-\frac{1}{2}}e^{x^2}2x$$